Lesson 26. Applications of Integration: Probability

1 Definitions

- A **random variable** is a variable that takes on its values by chance
  - One perspective: a random variable represents unknown future results
  - e.g. the time it takes to get a pizza delivered
- Let X and Y be a pair of random variables
  - e.g. the height and weight of an adult chosen at random
- The **joint density function** of X and Y is a function f of two variables that defines the probability (X, Y) are in some region D as follows:
- For example, when D is a rectangle $\{(X, Y) : a \leq X \leq b, c \leq Y \leq d\}$, then
- Probabilities aren't negative, so the joint density function must satisfy
- Probabilities are measured on a scale from 0 to 1, so
- The **expected value** of X is
- The **expected value** of Y is

2 Examples

Example 1. The joint density function for a pair of random variables X and Y

$$f(x,y) = \begin{cases} Cx(1+y) & \text{if } 0 \le x \le 1, 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of the constant C.
b. Find $P(X \le 1, Y \le 1)$.
c. Find $P(X + Y \le 1)$.

Example 2. The joint density function for a pair of random variables X and Y

$$f(x,y) = \begin{cases} 0.1e^{-0.5x-0.2y} & \text{if } x \geq 0,\ y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

a. Verify that f is a joint density function.
b. Find the expected values of X and Y.